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# Kinematic Modeling and Stable Control Law Designing for Four Mecanum Wheeled Mobile Robot Platform Based on Lyapunov Stability Criterion

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**Abstract:** Transportation in warehouses and production workshops is a matter of urgency today. Most warehouses arrange routes for circulation along the shelves, transportation vehicles will move on this road to perform the task of exporting or importing goods. Routes will be arranged to move in one direction because vehicles do not have enough space to turn around in cramped warehouses. This causes many difficulties in planning the trajectory for transportation vehicles, especially self-propelled vehicles. In order to have an appropriate transportation plan, it is necessary to solve many problems, including: reasonable transport equipment, sufficient number of devices, optimal route layout, algorithm of operation center for Positioning and Navigation of transportation equipment, This study proposes a method for transportation using an omnidirectional automated guided vehicle (AGV). The AGV's omnidirectional mobility is supported by the mecanum wheels, so vehicles can move in multiple directions on the road without turning, even at a junction or an intersection. This study consists of two parts, the first part focuses on kinematic modeling for mecanum wheels and extends to robot's platform using four mecanum wheels. Part two proposes a diagram to calculate the errors of the robot compared to a reference tracking line, design a control law based on the Lyapunov stability criterion. The stability of the control law is verified and confirmed by simulation on Matlab environment.

**Keywords:** Mecanum Wheels, Omni Directional Mobile Platform, Lyapunov Stability, Line Tracking Robots, AGV

## 1. Introduction

Mecanum wheel, sometimes called the Swedish wheel or Ilon wheel, was invented by Bengt Erland Ilon in 1973. In this design, there are many of free rollers attached to the hub. These rollers typically each have an axis of rotation at 45° to the wheel plane and at 45° to the axle line [1]. Each Mecanum wheel is driven by a separate motor for an omnidirectional movement on a plane [2]. The advantage of AGV (Automated Guided Vehicle) using Mecanum wheels is that there is no steering mechanism. Figure 1 [11] present the picture and the drawing model of a Mecanum wheel.

A robot moving in a plane will have a maximum of 3 degrees of freedom (DoFs). So that, the AGV must have at least three wheels [4-10]. In the field of industrial, four Mecanum wheels AGV is widely used. There are many kinds of 4-wheeled Mecanum configuration for a mobile platform. However, in order to implement an omnidirectional AGV, it is necessary to select a properly configuration. To ensure the omni-directional movement ability, the axes of bottom roller of any three wheels have to intersect at two points [3]. There are two configurations that are selected to archive omnidirectional motion, they are illustrated in Figure 2. In Figure 2, the axes of bottom roller of any three wheels intersect with other at two distinguishing points. But especially in right hand side of Figure 2, if the center of four wheels forms a square, the axes of bottom roller of any three wheels intersect at only one point. In this case, the Jacobi matrix [3] will be singular and the platform is no longer an omnidirectional system. Jingyang et al. [13] designed a prototype of omnidirectional mobile robot, named Savvy, and a software system framework for this platform based on ROS environment, SLAM. Taheri et al. [14] introduced omnidirectional Mecanum wheels and its kinematic relation of a mobile platform using four Mecanum wheels. The study also obtained the experimental of 8 different motions without the change of its orientation. Doreftei et al. [15] presented a literature review of practical application for mobile robot using special wheels. According to the study of Doreftei, mobile equipment with omnidirectional property can be moved in widely environment, such as: factory workshops, warehouses, hospitals, etc. Li et al. [16] proposed a method for modeling the Mecanum wheel platform. In other to verify the modeling results, a virtual model is established in SolidWorks and a virtual prototype used for simulation is conducted in RecurDyn software. Daniil et Vasily [17] discussed about the design feature of a Mecanum wheel platform. This study also mentioned to the forward and revert kinematics problem for this kind of wheel. The results of mathematics model are proved by using Mobile Robotics Simulation Toolbox package, an additional package of the Matlab Simulink.

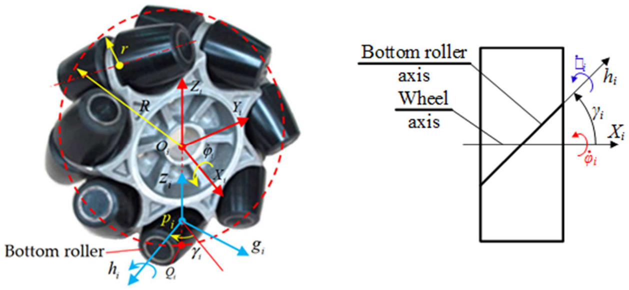


***Figure 1.*** *Mecanum wheel and its basic parameter.*

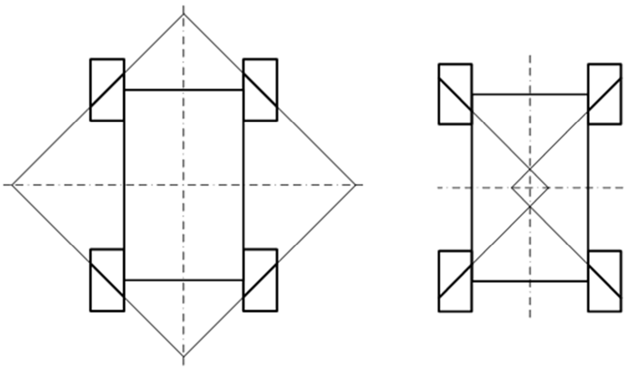

***Figure 2.*** *Two optimal configurations for four Mecanum wheeled platform.*

This study will focus on the kinematic analysis for the Mecanum wheel and extend to the robot model using four Mecanum wheels. Then, using the modeling results for the four Mecanum wheeled platform combined with the Lyapunov stability criterion to design the control law for the robot to follow the given reference trajectory.

## 2. System Modeling

### *2.1. Modeling of Four Mecanum Wheeled Platform*

The Mecanum wheel is modeled as shown in Figure 1. Coordinate system $O_iX_iY_iZ_i$ is attached to $i$-th wheel, where $x_i$ is the axis of rotation of this wheel, $O_i$ is coincided with the center of $i$-th wheel. $Z_i$ is the vertical axis passing through $O_i$ in the upward direction. $\overrightarrow{X_i}$, $\overrightarrow{Y_i}$ and $\overrightarrow{Z_i}$ follow the right-hand rule. In addition, $\overrightarrow{h_i}$ is the axis of rotation of the roller. $\gamma_i$ denotes the angle from $\overrightarrow{X_i}$ to $\overrightarrow{h_i}$. $R$ and $r$ are wheel and roller radius, respectively. $\dot{\varphi}_i$ and $\dot{\phi}_i$ are angular rotational speed about $\overrightarrow{X_i}$ of $i$-th wheel and $\overrightarrow{h_i}$ of its roller, respectively. Where $i = 1..4$ is the wheel's index. In addition, $P_ig_ih_iz_i$ is the frame that coincides with the roller in contact with the ground, $g_i$, $h_i$ and $z_i$ also follow the right-hand rule. Let $P_i$ be the center of the roller in contact with the ground, $Q_i$ be the point of contact of the roller and the ground. According to the above assumption, we have $\overrightarrow{P_iQ_i}$ parallel to $\overrightarrow{Z_i}$, but in the opposite direction.

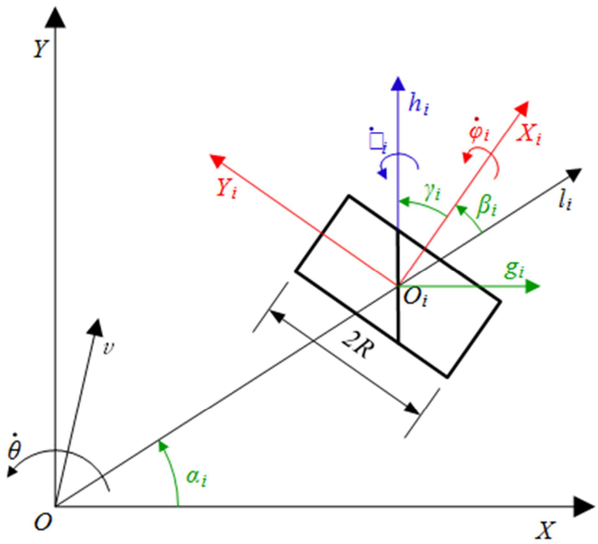


***Figure 3.*** *A Mecanum wheel's model.*

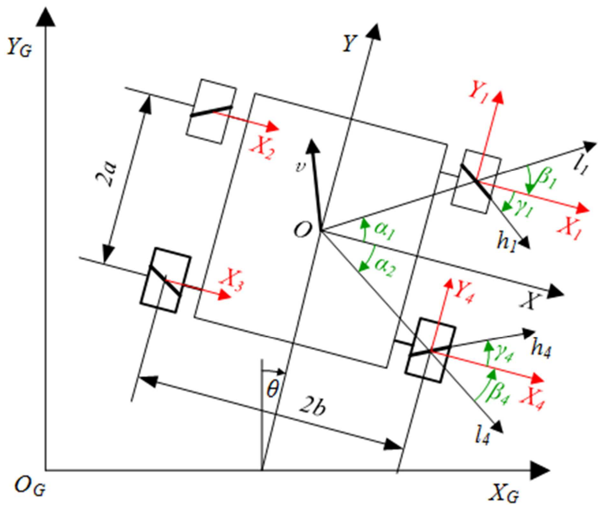


***Figure 4.*** *Four-wheeled mobile platform with Mecanum wheels modeling seen from stationary coordinate.*

Let $\overrightarrow{v_{Oi}}$ be the absolute velocity of the point $O_i$, $\overrightarrow{v_{pi}}$ be the absolute velocity of the point $P_i$, $\overrightarrow{v_{OP}}$ be the relative velocity of $O_i$ seen from $P_i$, we have:

$$\overrightarrow{v_{Oi}} = \overrightarrow{v_{pi}} + \overrightarrow{v_{OP}} \tag{1}$$

Denoted $\overrightarrow{\omega_{Oi}}$ and $\overrightarrow{\omega_{pi}}$ are the angular rotational velocity of the $i$-th wheel and its roller respectively in the fixed coordinate system, they can be calculated as follows:

$$\overrightarrow{\omega_{Oi}} = \dot{\theta}\overrightarrow{Z_i} + \dot{\varphi}_i\overrightarrow{X_i} \text{ and } \overrightarrow{\omega_{pi}} = \overrightarrow{\omega_{Oi}} + \dot{\phi}_i\overrightarrow{h_i} \quad (2)$$

The values of linear velocities $\overrightarrow{v_{pi}}$ is determined by multiplying the equations (2) with the level arm $\overrightarrow{Q_iP_i}$ as follow:

$$\overrightarrow{v_{pi}} = \overrightarrow{\omega_{pi}} \times \overrightarrow{Q_iP_i} = \left(\dot{\theta}\overrightarrow{Z_i} + \dot{\varphi}_i\overrightarrow{X_i} + \dot{\phi}_i\overrightarrow{h_i}\right) \times r\overrightarrow{Z_i} = -r\left(\dot{\varphi}_i\overrightarrow{Y_i} - \dot{\phi}_i\overrightarrow{g_i}\right) \quad (3)$$

Similarly, $\overrightarrow{v_{OP}}$ is obtained:

$$\overrightarrow{v_{OP}} = \overrightarrow{\omega_{Oi}} \times \overrightarrow{P_iO_i} = (R-r)\left(\dot{\theta}\overrightarrow{Z_i} + \dot{\varphi}_i\overrightarrow{X_i}\right) \times \overrightarrow{Z_i} = (r-R)\dot{\varphi}_i\overrightarrow{Y_i} \quad (4)$$

Where $R$ and $r$ are wheel and roller radius respectively. Then, $\overrightarrow{v_{oi}}$ is calculated by adding the two above equations together:

$$\overrightarrow{v_{oi}} = -r\left(\dot{\varphi}_i\overrightarrow{Y_i} - \dot{\phi}_i\overrightarrow{g_i}\right) + (r-R)\dot{\varphi}_i\overrightarrow{Y_i} = r\dot{\phi}_i\overrightarrow{g_i} - R\dot{\varphi}_i\overrightarrow{Y_i} \quad (5)$$

On the other hand, let $\vec{v}$ be velocity of centre point $O$ seen from the ground, we have the relationship between $\overrightarrow{v_{oi}}$, $\vec{v}$ and $\dot{\theta}$: $\overrightarrow{v_{oi}} = \vec{v} + \dot{\theta}\left(\vec{Z} \times \overrightarrow{l_i}\right)$. So that:

$$r\dot{\phi}_i\overrightarrow{g_i} - R\dot{\varphi}_i\overrightarrow{Y_i} = \vec{v} + \dot{\theta}\left(\vec{Z} \times \overrightarrow{l_i}\right) \quad (6)$$

Multiply the scalar by $\overrightarrow{h_i}$ on both sides of the above equation, we get:

$$r\dot{\phi}_i\overrightarrow{h_i}.\overrightarrow{g_i} - R\dot{\varphi}_i\overrightarrow{h_i}.\overrightarrow{Y_i} = \overrightarrow{h_i}.\vec{v} + \dot{\theta}\overrightarrow{h_i}.\left(\vec{Z} \times \overrightarrow{l_i}\right) \quad (7)$$

In fact $\overrightarrow{g_i}.\overrightarrow{h_i} = 0$; and $\overrightarrow{Y_i}.\overrightarrow{h_i} = \sin\gamma_i$. Therefore:

$$-R\dot{\varphi}_i\sin\gamma_i = \overrightarrow{h_i}.\vec{v} + \dot{\theta}\overrightarrow{h_i}.\left(\vec{Z} \times \overrightarrow{l_i}\right) \quad (8)$$

In the configuration of the mobile robot shown in Figure 4, because $\overrightarrow{X_i}$ is always set to parallel with $\vec{X}$, so $\alpha_i + \beta_i = 0$. Apply this equation, vector $\overrightarrow{h_i}$ seen from stationary coordinate system is described as below:

$$\overrightarrow{h_i} = [\cos(\alpha_i + \beta_i + \gamma_i)\,;\sin(\alpha_i + \beta_i + \gamma_i)\,;0] = [\cos\gamma_i\,;\sin\gamma_i\,;0] \quad (9)$$

And vector $\overrightarrow{l_i}$ is also: $\overrightarrow{l_i} = [l_i\cos\alpha_i\,; l_i\sin\alpha_i\,;0]$

The results of scalar product and cross product between $\overrightarrow{h_i}$, $\vec{v}$, $\vec{Z}$ and $\overrightarrow{l_i}$ are recognized as follows:

$$\overrightarrow{h_i}.\vec{v} = v_x\cos\gamma_i + v_y\sin\gamma_i \text{ and } \vec{Z} \times \overrightarrow{l_i} = [-l_i\sin\alpha_i\,; l_i\cos\alpha_i\,;0] \quad (10)$$

Substitute equations (10) into formula (8), we get:

$$-R\dot{\varphi}_i\sin\gamma_i = v_x\cos\gamma_i + v_y\sin\gamma_i - \dot{\theta}l_i\sin\alpha_i\cos\gamma_i + \dot{\theta}l_i\cos\alpha_i\sin\gamma_i \quad (11)$$

Because $R\sin\gamma_i$ is never equal to zero, divide both sides of the equation (11) by $R\sin\gamma_i$, the $i$-th wheel angular rotational speed $\dot{\varphi}_i$ is determined through the robot's linear velocity and angular velocity as in the below equation:

$$\dot{\varphi}_i = -\frac{1}{R_i}J_i\begin{bmatrix} v_x & v_y & \dot{\theta} \end{bmatrix}^T \quad (12)$$

In which $J_i = [\cot\gamma_i \quad 1 \quad -l_i\sin\alpha_i\cot\gamma_i + l_i\cos\alpha_i]$ is the $i$-th row of Jacobian matrix $J$. The values of the angular constants $\alpha_i$ and $\gamma_i$ are described as equation (13) and (14):

$$\gamma_2 = \gamma_4 = -\gamma_1 = -\gamma_3 = 45^0 \quad (13)$$

$$\alpha_1 \in (0;90^0);\ \alpha_1 \in (90^0;180^0);\ \alpha_3 \in (-90;-180^0);\alpha_4 \in (0;-90^0) \quad (14)$$

Fully kinematic equation for the four Mecanum wheeled robot is described as below:

$$\begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \\ \dot{\varphi}_4 \end{bmatrix} = -\frac{1}{R}\begin{bmatrix} -1 & 1 & a+b \\ 1 & 1 & -a-b \\ -1 & 1 & -a-b \\ 1 & 1 & a+b \end{bmatrix}\begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix} \quad (15)$$

Equation given above is the inverse kinematic equation of the four-wheeled robot, in which the angular rotational speed of each wheel is calculated according to the linear and angular velocity of the robot in a fixed coordinate system.

### *2.2. Line Tracking's Deviations Modeling*

In this section, we will propose a mathematical model for a mobile robot to follow a given reference line. Tracking line is assumed as a curve in $O_GX_GY_G$ coordinate system. The setting reference point $R(x_R, y_R)$ is assumed to be moving along the reference orbit with linear velocity $\overrightarrow{v_r}$, the absolute value $|\overrightarrow{v_r}|$ is set to be constant. Mobile platform with parameters as mentioned in the previous section, the control point $C(x_C, y_C)$ will track to the reference point $R$ with constraint that the $\vec{Y}$ axis is always tangent to the reference curve.

In most cases, point C is chosen to coincide with the robot's center of gravity. In this study, the control point C is chosen to coincide with the geometric center point $O$ of the Mecanum platform. We have the kinematics equation of point $C$ in a fixed coordinate system as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\tau}_C \end{bmatrix} = \begin{bmatrix} \cos\tau_C & 0 \\ \sin\tau_C & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_C \\ \omega_C \end{bmatrix} \quad (16)$$

Where $v_C$ and $\omega_C$ are the linear and angular velocity of the control point $C$. On the other hand, we also have the kinematic equation of the reference point $R$ in the fixed coordinate system as follows:

$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\tau}_R \end{bmatrix} = \begin{bmatrix} \cos\tau_R & 0 \\ \sin\tau_R & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_R \\ \omega_R \end{bmatrix} \quad (17)$$

The position deviation between C and R can be considered as three components as shown in Figure 5, including: Longitudinal deviation $e_1$, transverse deviation $e_2$ and angular deviation $e_3$. These errors can be represented in matrix form as follows:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\tau_C & \sin\tau_C & 0 \\ \sin\tau_C & -\cos\tau_C & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_R - x_C \\ y_R - y_C \\ \tau_R - \tau_C \end{bmatrix} \tag{18}$$

Taking the derivative of both sides of the above equation, we can get:

$$\begin{bmatrix} \dot e_1 \\ \dot e_2 \\ \dot e_3 \end{bmatrix} = \begin{bmatrix} -1 & -e_2 \\ 0 & e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v_C \\ \omega_C \end{bmatrix} + \begin{bmatrix} v_R \cos e_3 \\ -v_R \sin e_3 \\ \omega_R \end{bmatrix} \tag{19}$$

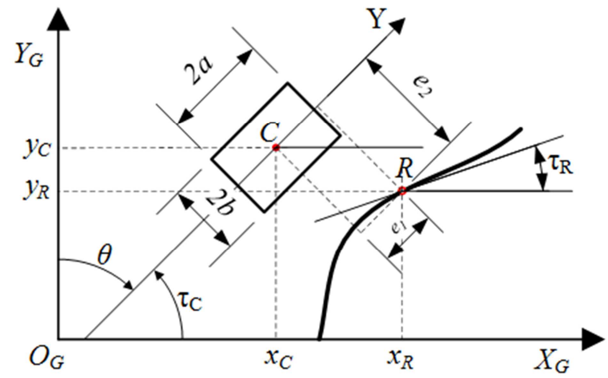


***Figure 5.*** *Diagram of deviations calculation for line tracking.*

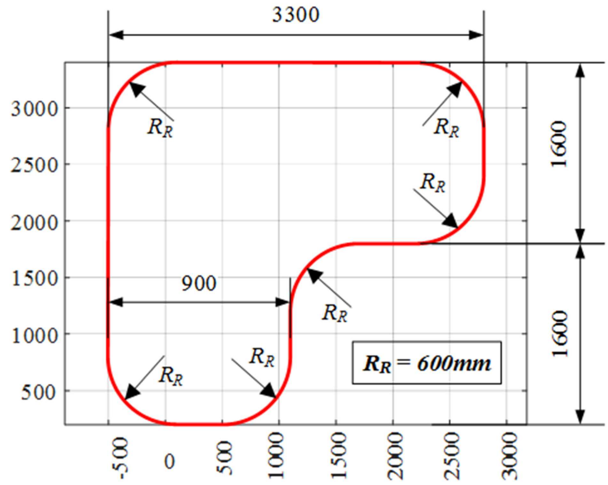


***Figure 6.*** *Reference line's parameters used in simulation.*

## 3. Control Law Designing

The objective of this section is to design a stable control law to ensure that the robot moves along the reference line, satisfying the moving conditions in Section II. B. The control object here is a robot with four Mecanum wheels. Here we only consider the kinematic relationship without regard to the dynamical parameters.

As analyzed above, linear velocity $v_C$, angular velocity $\omega_C$, moving direction $\theta$ of platform and the angular rotational speed of each Mecanum wheels are presented in equation (15). From Figure 5 we can recognize that $\tau_C = 90^0 + \theta$ and $\dot\theta = \dot\tau_C$. Let choose Lyapunov function as follows:

$$L = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + k_1(1 - \cos e_3) \tag{20}$$

Noted that $L \geq 0\ \forall(e_1, e_2, e_3)$ and $L = 0$ only when $(e_1, e_2, e_3) = (0,0,0)$.

Taking the derivative of the equation (20) versus time, we get:

$$\dot L = e_1\dot e_1 + e_2\dot e_2 + k_1\dot e_3 \sin e_3 \tag{21}$$

Combine equations (20), (18) and (19), we get:

$$\dot L = e_1(-v_C + v_R\cos e_3)) + k_1 \sin e_3 \left(\omega_R - \omega_C - \frac{e_2 v_R}{k_1}\right) \tag{22}$$

If we choose $v_C$ and $\omega_C$ as below equations:

$$\begin{cases} v_C = v_R \cos e_3 + k_2 e_1 \\ \omega_C = \omega_R - \frac{1}{k_1}(e_2 v_R - k_3 \sin e_3) \end{cases} \tag{23}$$

Then the equation (22) is simplified as following equation:

$$\dot L = -k_2 e_1^2 - k_3 \sin^2 e_3 \leq 0, \forall k_2, k_3 > 0 \tag{24}$$

According to Lyapunov stability criterion [12], robot platform will be stable with this control law.

## 4. Simulation Results

The simulation is done by using Matlab environment in two cases. First case is the simulation with assume that all the tracking line parameters are known and second case is the simulation in the situation of some parameters are unknown.

### *4.1. Simulate Results in Case the Tracking Curve's Parameters are Known*

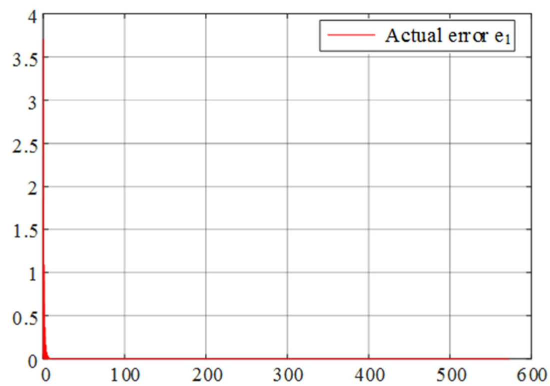


***Figure 7.*** *Simulation result of longitudinal deviation.*

The control parameters are: $k_1 = 1; k_2 = 10$ and $k_3 = 2$. Layout of the tracking line used in this simulation is shown in Figure 6, and the kinematic parameters of the robot are listed as below: Platform's half-width $b = 120\ (mm)$, platform's half-length $a = 100(mm)$, wheel radius $R = 45(mm)$, roller radius $r = 10\ (mm)$, reference curve radius $R_R = 600\ (mm)$, reference linear velocity $v_r = 10\ (mm/s)$ and sampling time $t_s = 0.1(s)$. Assume that robot's initial position has an deviation relative to reference point $R$ as: $e_1 = 0\ (mm); e_2 = 10\ (mm)$ and $e_3 = pi/18\ (rad)$. Figure 7 to Figure 9 show deviations of simulation result with sampling time $t_s = 0.1$ second. The vertical axis of each chart represents the value of the error ($mm$ for $e_1$ and $e_2$, and $rad$ for $e_3$), the horizontal axis represents the simulation time in second. The time it takes for the robot to complete this orbit is about 572.6 seconds. From the charts it can be seen that, due to the initial error of $e_2$ and $e_3$, the errors are somewhat fluctuating at the beginning, after about 15 seconds the steady-state value is approximately 0 mm for $e_1$ and $e_2$ or 0 rad for $e_3$. Figure 10 to Figure 12 show the simulation results of deviations for period from 0 to 15 seconds only.

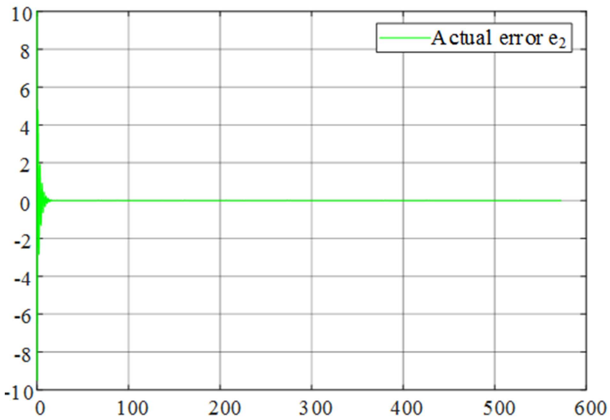


***Figure 8.*** *Simulation result of transverse deviation.*

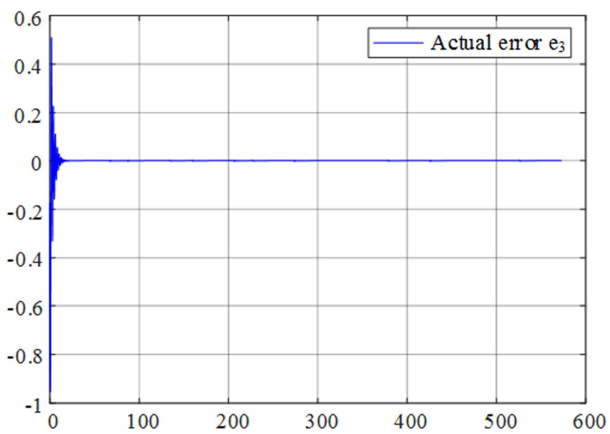


***Figure 9.*** *Simulation result of angular deviation.*

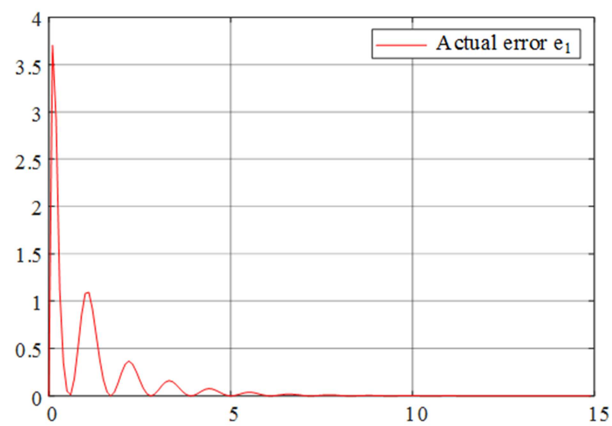


***Figure 10.*** *Simulation result of longitudinal deviation.*

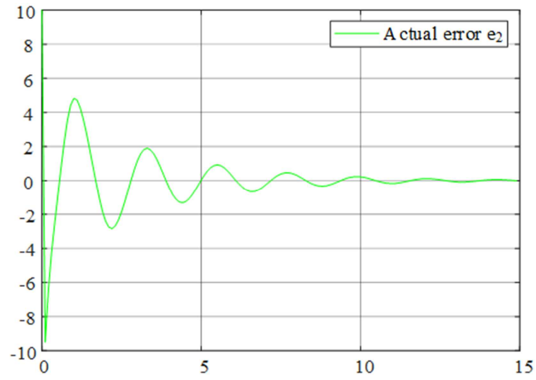


***Figure 11.*** *Simulation result of transverse deviation.*

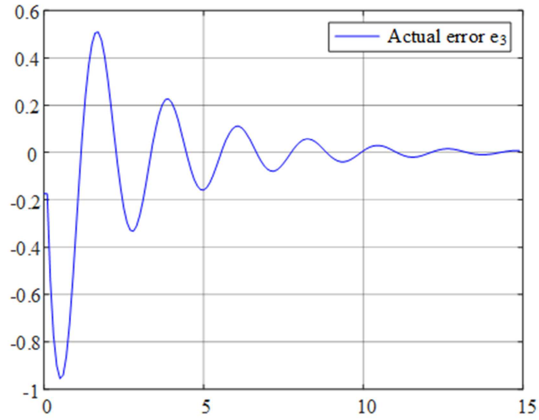


***Figure 12.*** *Simulation result of angular deviation.*

### *4.2. Simulate Results in Case the Tracking Curve's Parameters are Unknown*

In fact, robots moving in warehouses and production workshops will follow some reference curve such as magnetic line, induction line or laser virtual line. When moving along these lines, parameters such as $\omega_R$ and $e_1$ cannot be determined. In addition, the error value $e_3$ may or may not be determined depending on type and number of sensors used. Here we will investigate in both cases of deviation value $e_3$: can and cannot be determined. The kinematics and control parameters are unchanged. Figure 13 and Figure 14 show simulation results in case $e_3$ can be determined, Figure 15 show simulation result in case the value of $e_3$ is not available.

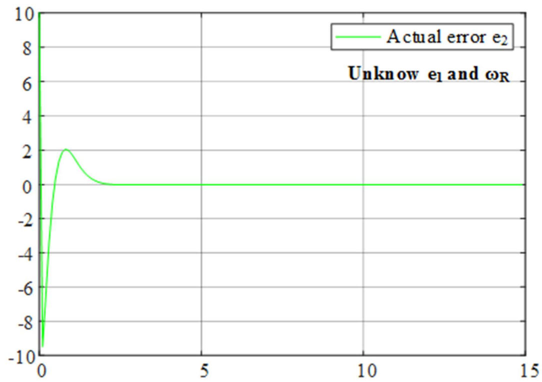


***Figure 13.*** *Simulation result of transverse deviation.*

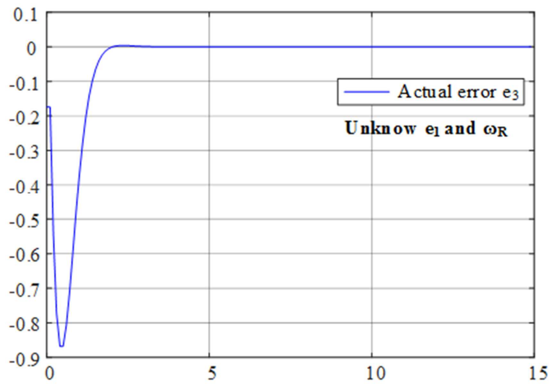


***Figure 14.*** *Simulation result of angular deviation.*

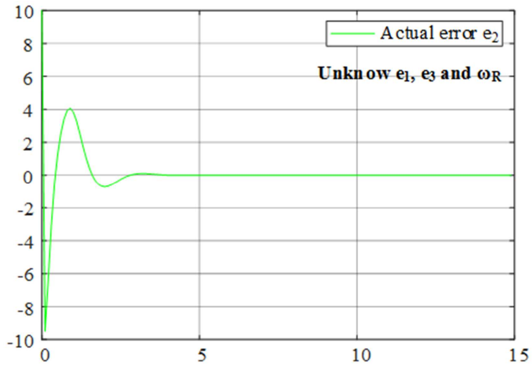


***Figure 15.*** *Simulation result of transverse deviation.*

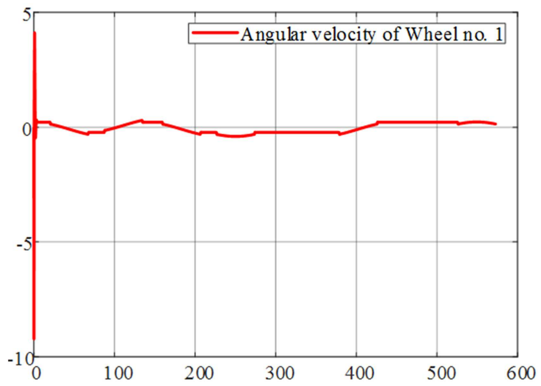


***Figure 16.*** *Simulation result for angular velocity of wheel No. 1.*

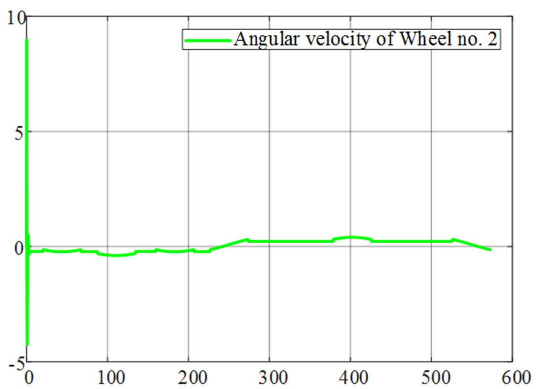


***Figure 17.*** *Simulation result for angular velocity of wheel No. 2.*

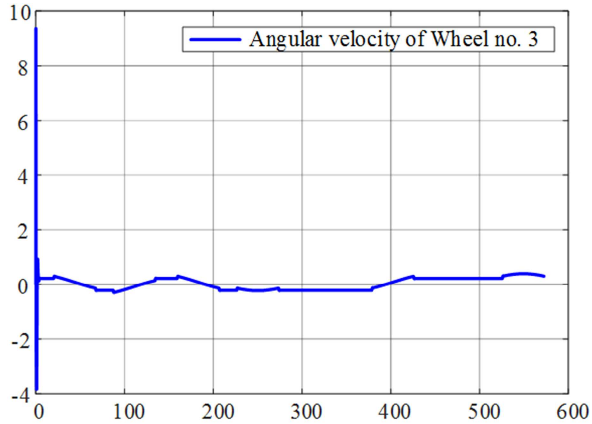


***Figure 18.*** *Simulation result for angular velocity of wheel No. 3.*

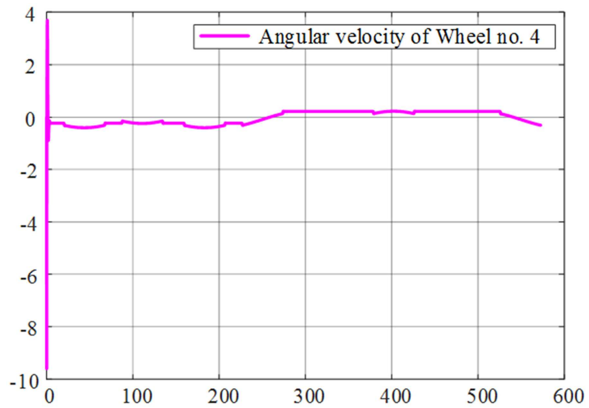


***Figure 19.*** *Simulation result for angular velocity of wheel No. 4.*

The angular rotational velocity of each wheel in the case of unknown value of $e_3$ based on the relationship in equation (22) is shown as Figure 16 to Figure 19. The vertical axis is the value of the wheel angular velocity in rad/s, the horizontal axis is the time of simulation in second.

## 5. Conclusions

This study focuses on modeling the kinematics in general for only one Mecanum wheel, then extended to mobile robot using four Mecanum wheels. Based on this result, the research continues with a specific application of controlling a four Mecanum wheeled robot to follow a given reference line. This study does not refer to the type of line or the method of measuring and determining the error by real sensors. This research also provides a diagram to calculate the tracking deviation based on the position of the robot relative to the desired curve, and also proposes a control law to ensure that the robot moves along the line with a constant speed based on the Lyapunov stability criterion. The simulation results on Matlab environment have verified the correctness of the mathematical model and the stability of the control law.

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